The Tensile Strength and Ductility of Continuous Fibre Composites

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The plastic instability approach has been applied to the tensile behaviour of a continuous fibre composite. It is shown that the combination of two components with different strengths and degrees of work-hardening produces a new material with a new degree of work-hardening, which may be determined by the present analysis. Expressions for the elongation at rupture and the strength of a composite have been obtained and the results of the calculation are compared with some experimental data.

1. Introduction

The starting point for many treatments, published in the last four or five years, of the strength of continuous fibre composites, is the relationship analysed by Kelly and Tyson [1],

$$\sigma_* = (1 - V_f)\,\sigma_*'' \quad \text{at } 0 \leqslant V_f \leqslant V_{f*} \qquad (1)$$

$$\sigma_* = V_f\sigma_*' + (1 - V_f)\sigma_m'' \text{ at } V_{f*} \leqslant V_f \leqslant 1 \qquad (2)$$

Here V_f is volume fraction of fibres, σ_*' and σ_*'' are ultimate tensile strength of fibres and matrix respectively, σ_m'' is the matrix stress at the moment when the fibres are broken and $\sigma_* = \sigma_{*\min}$ at $V_f = V_{f*}$.

These formulae are valid for brittle fibres only; the line AOB in fig. 1 corresponds to that case.

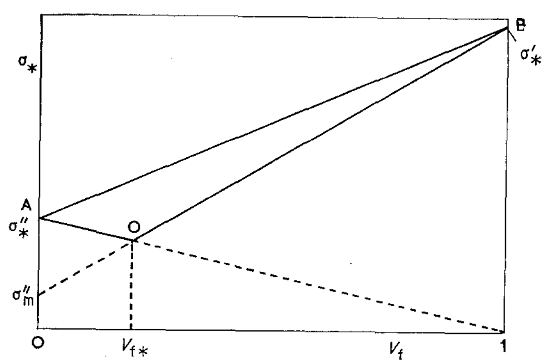

Figure 1 Schematic diagram for strength – volume fraction dependence.

However, most results obtained for metal-metal fibre composites have been interpreted as a confirmation of the so-called law of mixtures, because usually they have not shown a minimum point like O in fig. 1. It would then seem that the relation between composite strength and volume fraction was approximately linear. It will be shown here that the application of plastic instability theory to fibre composites explains the reason for this deviation from the behaviour predicted by Kelly and Tyson. Furthermore, experimental data are compared with the predictions of the theory.

2. Plastic Instability of a Composite Material

We considered the case when both the matrix and the reinforcing fibre have some ductility. The ultimate tensile strength of a homogeneous, ductile material is determined by the condition of plastic instability [2]. The nominal stress $\sigma = Q/A_0$ reaches the maximum value σ_* when a rod achieves an unstable state, that is necking begins. (Q is load, A_0 is initial value of cross-sectional area A). If the stress/strain curve expressed in true co-ordinates,

$$s = Q/A,\ \epsilon = \ln l/l_0$$

is approximated by a power function

$$\epsilon = (s/s_*)^n \qquad (3)$$

where s_* and n are constants, then assuming

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incompressibility of material, we obtain the equation for conventional stress/true strain in the form

$$\sigma = s_* \, \epsilon^{1/n} \exp(-\epsilon) \, . \tag{4}$$

The maximum nominal stress σ_* is reached at $\epsilon = \epsilon_*$, which is determined by the condition $d\sigma/d\epsilon = 0$. This condition and equation 4 give

$$\epsilon_* = 1/n \, .$$

Therefore all the constants in approximation 3 are expressed by values which are obtained in a tensile test:

$$n = 1/\epsilon_*, \; s_* = \sigma_* \epsilon_*^{-\epsilon_*} \exp \epsilon_* \, .$$

Now we can rewrite expression 4 in the form

$$\sigma = \sigma_* \, (\epsilon/\epsilon_*)^{\epsilon_*} \exp(\epsilon_* - \epsilon) \, . \tag{5}$$

Usually ϵ_* is approximately equal to the strain at rupture although this is not true for all materials. In general the neck develops after the critical point is reached, on the falling part of a stress/strain curve. This process is not described by the present approach.

We shall now consider the behaviour of a fibre composite material. We assume that the bond between fibre and matrix is an ideal one. The necking of any one component is impossible without necking of the composite as a whole. The second assumption is that expression 3 is valid to some strain beyond ϵ_*. The stress/strain curve of the matrix is characterised by the constants σ_*'' and ϵ_*'', and similarly for the fibres by σ_*' and ϵ_*'. If the stress and strain of the composite are σ and ϵ, then we have

$$\sigma = V_f \, \sigma' + (1 - V_f) \, \sigma'' \tag{6}$$

$$\epsilon = \epsilon' = \epsilon'' \tag{7}$$

$$\sigma = V_f \, \sigma_*' \, (\epsilon/\epsilon_*')^{\epsilon_*'} \exp(\epsilon_*' - \epsilon) + (1 - V_f) \, \sigma_*'' \, (\epsilon/\epsilon_*'')^{\epsilon_*''} \exp(\epsilon_*'' - \epsilon) . \tag{8}$$

Equation 8 which gives the stress/strain curve of the composite is a consequence of equations 5 to 7.

Differentiating the expression 8 with respect to ϵ we obtain an equation for the determination of the critical strain ϵ_* of the composite,

$$V_f \, \sigma_*'(\epsilon/\epsilon_*')^{\epsilon_*'} \, (\epsilon_*'/\epsilon'_* - 1) \exp \epsilon_*' + (1 - V_f) \, \sigma_*'' \, (\epsilon/\epsilon_*)^{\epsilon_*''} (\epsilon_*''/\epsilon_* - 1) \exp \epsilon_*'' = 0 \, . \tag{9}$$

This equation is non-linear with respect to ϵ_*, but it is linear in V_f. Hence it is more convenient to use the following form:

$$V_f = \frac{1}{1 + \beta \, \dfrac{\epsilon_* - \epsilon_*'}{\epsilon_*'' - \epsilon_*} \, \epsilon_*^{\epsilon_*' - \epsilon_*''}} \tag{10}$$

where

$$\beta = \frac{\sigma_*'}{\sigma_*''} \, \frac{\epsilon_*''^{\epsilon_*''}}{\epsilon_*'^{\epsilon_*'}} \, \frac{\exp \epsilon_*'}{\exp \epsilon_*''} \, . \tag{11}$$

It can be shown that $\epsilon_*' < \epsilon_* < \epsilon_*''$, if $\epsilon_*' < \epsilon_*''$ and $0 < V_f < 1$, i.e. the critical deformation of the composite is larger than that for separate fibres. As we have assumed above that the strength of the fibre-matrix interface is sufficient to prevent the fibre necking without necking of the composite as a whole, the result obtained implies that achievement of the maximum on the stress/strain curve of the fibre at $\epsilon = \epsilon_*'$ is not accompanied by the beginning of composite necking. In short, the more stable matrix restrains the less stable fibre. The stress/strain curve of the fibre follows equation 5 up to the moment of necking of the composite, i.e. the homogeneous stable strain of the fibre reaches a value $\epsilon_* > \epsilon_*'$. This situation is illustrated by fig. 2. The falling part of the curve

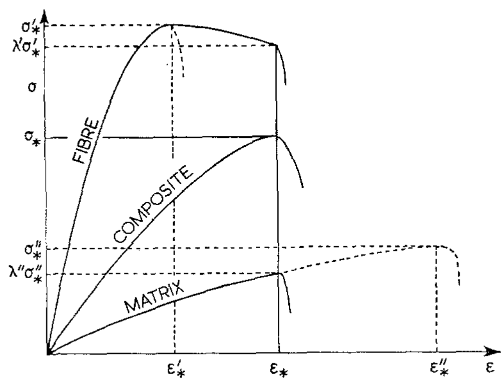


Figure 2 Schematic diagrams for fibre and matrix combined in composite and for composite. The dotted lines correspond to behaviour of fibre and matrix tested separately.

for separate fibres is shown by the dotted line, the solid line corresponds to the behaviour of the fibre in the composite. It is very important to note that at the critical composite strain $\epsilon = \epsilon_*$ the composite stress has achieved a maximum value σ_*, but the fibre stress has passed beyond the maximum point; also on the portion of the curve between ϵ_*' and ϵ_* there is some loading of the hardening matrix and some unloading of the fibre.

We are now able to determine the ultimate stress of a composite, σ_*, which corresponds to the critical value of deformation $\epsilon = \epsilon_*$. From equation 8 we obtain

$$\sigma_* = V_f \, \lambda' \sigma_*' + (1 - V_f) \, \lambda'' \sigma_*'' , \quad (12)$$

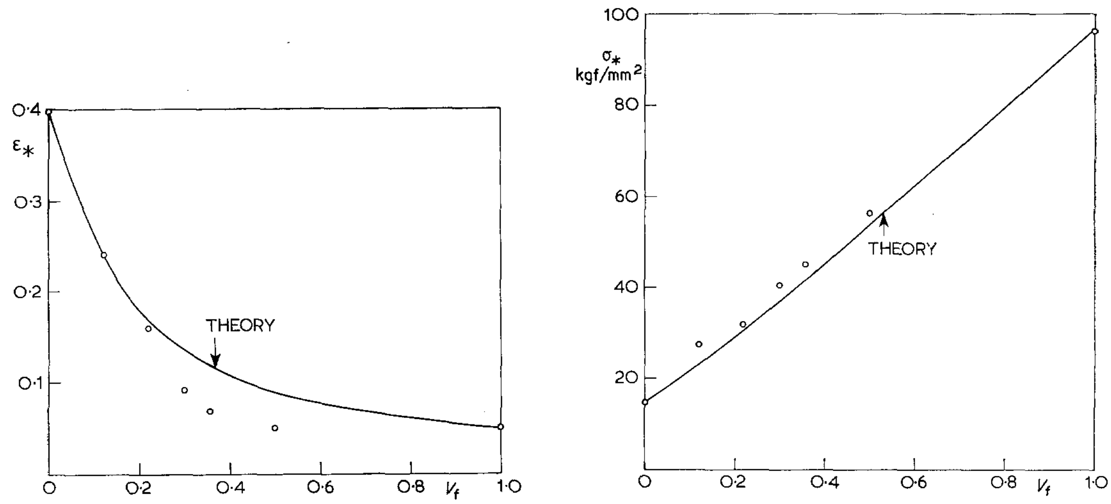
where

$$\lambda' = (\epsilon_*/\epsilon_*')^{\epsilon_*'} \exp(\epsilon_*' - \epsilon_*) ,$$
$$\lambda'' = (\epsilon_*/\epsilon_*'')^{\epsilon_*''} \exp(\epsilon_*'' - \epsilon_*) .$$

Formula 12 is illustrated in fig. 2. It gives the dependence of the composite's ultimate strength on the ultimate strength and critical deformation of matrix and fibre. It is easy to show in the case $\epsilon_*' = \epsilon_*''$, that the composite strengths correspond to line AB in fig. 1. All the curves for $\epsilon_*' < \epsilon_*''$ lie lower than AB. Strictly, this analysis may not be applied to the case of brittle fibres because of assuming the power function approximation, 3. However, the physical situation is clear and it is possible to conclude that the whole family of the curves lies within AOB.

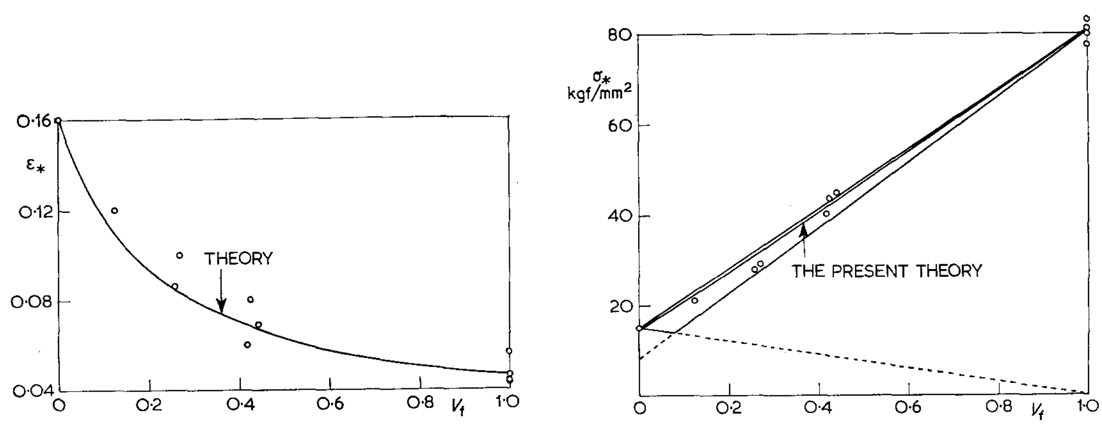
3. Comparison with Experiments

The comparison of the above calculations and experimental results obtained by A. Markov *et al* [3] for a monofilament Ni–W composite at a temperature of 400° C is given in figs. 3a and b. The results of calculations for the experimental data on strength and ductility of silver-stainless steel wire at room temperature obtained by H. Piehler [4] are shown in figs. 4a and b. The results for a Cu–Mo composite at room tempera-



(a) (b)

Figure 3 Critical strain (a) and the strength (b) of composite Ni–W at temperature 400° C as a function of volume fraction.



(a) (b)

Figure 4 Critical strain (a) and strength (b) of silver-stainless steel composite at room temperature as a function of volume fraction. Experimental data by H. R. Piehler [4].

ture by A. Kelly and W. R. Tyson [1] are compared with the theory in fig. 5.

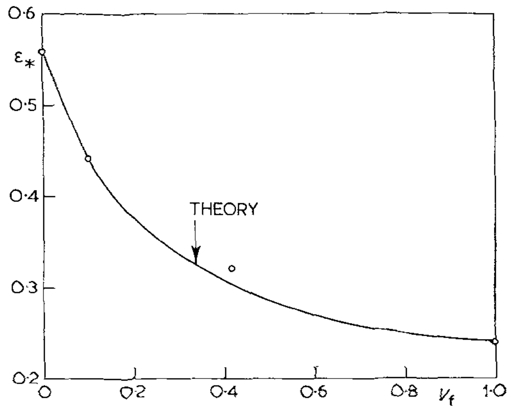


Figure 5 Critical strain of Cu–W composite at room temperature as a function of volume fraction. Experimental data by A. Kelly and W. R. Tyson [1].

The good agreement between theory and experiment in figs. 3a, 4a and 5 indicates that it is possible to use 3 to describe the behaviour of the individual components until a plastic instability of the composite commences in a tensile test.

It is clear that the best comparison may be obtained for the elongation at rupture because more precise calculation of the strength demands consideration of a complex stress state of composite form.

4. Conclusions

The present approach, based on the plastic instability theory predicts a more real behaviour of the components in composite materials. Good agreement with experimental data has been obtained for the elongation at rupture.

5. List of Symbols

V_f = volume fraction of fibres in composite.
$\epsilon', \epsilon'', \epsilon$ = true strain of fibre, matrix and composite.
s = true stress.
$\sigma', \sigma'', \sigma$ = nominal stress on fibre, matrix and composite.
$\sigma_*', \sigma_*'', \sigma_*$ = critical stress of fibre, matrix and composite (ultimate tensile strength).
$\epsilon_*', \epsilon_*''$ = critical strain of separate fibre and matrix.
ϵ_* = critical strain of composite.
Q = external load.
A = cross-sectional area.
A_0 = initial value of area.

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